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QUESTIONS AND DISCUSSIONS.

EDITED BY W. A. HURWITZ, Cornell University, Ithaca, N. Y.

DISCUSSIONS.

In the first discussion this month, Mr. Kazarinoff gives simple demonstrations of several theorems on conic sections in space, suggested by the properties of focal conics of a system of confocal quadrics. Without access to original sources or other adequate references, the author was led to these theorems by direct study of Figs. 1 and 2 of the article. Though Mr. Kazarinoff's theorems lack entire novelty it is hoped that the directness and simplicity of his demonstrations may aid in making them more widely known and admired; and that students of conics may thereby be incited to look into the generalizations of Chasles and Plücker.

The second discussion is of interest as affording an instance of the need of a theorem in mathematics for the purpose of answering a particular question in an applied science. In connection with a condition for stability of thermodynamic equilibrium of a fluid phase of a two-component body Professor Trevor is led to a property of homogeneous functions. The result is itself of interest, and suggests the possibility of generalizations. Probably the denominator $x_3 + x_4$ might be altered to any linear function of the variables with like results. Do similar theorems exist for more general types of transformation? What are the corresponding facts for other isobaric functions than homogeneous? Can the facts in the present and other cases be deduced in any simpler way?

Direct interpretations of imaginary intersections of geometric loci, by operations on the figure itself, have frequently aroused interest. Several such treatments exist for determining complex roots of quadratic and cubic equations. Mr. Mathews gives in the third discussion a unified treatment, by which a number of such questions can be resolved by the application of essentially a single idea—his *virtual image* of a circle with regard to a straight line.

I. DUPIN'S THEOREM.[1]

By D. C. KAZARINOFF, University of Michigan.

It is well known that the consideration of the family of confocal quadrics

$$\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}+\frac{z^2}{c^2+\lambda}=1 \quad (a^2>b^2>c^2), \qquad (F)$$

leads to the three focal conics:[2]

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[1] Read before the Minnesota Section of the Mathematical Association of America, May 31, 1919.

[2] C. Smith, *An elementary treatise on Solid Geometry*, 10th ed., London, 1905, pp. 144, 145.